Journal of Statistics and Computer Science
Vol. 2, Issue 1, 2023, pp. 47-53
https://doi.org/10.47509/JSCS.2023.v02i01.04

Some Methods of Constructions of Partially Balanced Designs

Shyam Saurabh
Tata College, Kolhan University, Chaibasa, India
Email: *shyamsaurabh785@gmail.com*

Abstract: The idea of near resolvability for balanced incomplete block design is extended to partially balanced designs and some such designs are obtained. Some series of group divisible designs, triangular designs and cyclic designs are obtained from α–resolvable group divisible designs, near resolvable triangular and near resolvable cyclic designs respectively. These designs are important classes of partially balanced incomplete block designs.

MSC: 62K10; 05B05

Keywords: Near resolvable designs; α–resolvable designs; Group divisible designs; Triangular designs; Cyclic designs; LDPC codes

1. Introduction

In this paper the idea of near resolvability for balanced incomplete block designs (BIBDs)/ $2-(v,k,\lambda)$ designs is extended to triangular and cyclic designs; and some such designs are obtained. Further some series of partially balanced incomplete block designs including semi–regular group divisible (SRGD), triangular and cyclic designs are obtained from α–resolvable SRGD, near resolvable triangular and cyclic designs respectively.

Some methods of constructions of near resolvable $2-(v,k,\lambda)$ designs along with their existence and applications may be found in Furino (1995), Furino *et al.* (1996), Abel *et al.* (2008) and Abel (2017), among others. Trung (1999) used classical inversive planes of even order to construct a class of $2-(2^{2n}+1, 2^n, 2^n-1)$ near resolvable designs, in which any two blocks have at most two treatments in common. Bassalygo and Zinoviev (2016) showed the equivalence between a resolvable $2-(v,k,1)$ and a near resolvable $2-(v,k-1,k-2)$ design. A table of near resolvable $2-(v,k,\lambda)$ designs under the range of $15 \le v \le 200$; $3 \le k \le 97$ may be found in Furino *et al.* (1996). Some constructions of combinatorial matrices from affine resolvable and near resolvable designs may be found in Saurabh (2022).

For details on partially balanced designs, see Clatworthy (1973), Raghavarao (1988), Bagchi (2004), Raghavarao and Padgett (2005), Dey (2010), Arasu *et al.* (2013), Saurabh *et al.* (2021a), Saurabh and Sinha (2023) and references therein. *SRX*, *TX* and C*X* denote a SRGD design, triangular design and cyclic design with number *X* respectively as listed in Clatworthy (1973).

The rest of the paper is organized as follows. Section 2 contains some basic definitions and terminologies. Section 3 describes some constructions methods of SRGD, triangular and cyclic designs from α –resolvable SRGD, near resolvable triangular and cyclic designs respectively. Section 4 contains some examples of near resolvable and related designs whereas Section 5 describes future prospects and some applications of these designs.

2. Definitions

2.1 Near resolvable design

A block design $D(v,b,r,k)$ is said to be *near resolvable* if its blocks can be partitioned into v classes such that for each treatment θ of D there is precisely one class which does not contain θ in any of its blocks and each class contains $v-1$ distinct treatments. Such classes are known as partial resolution classes. The necessary conditions for the existence of such designs are $v \equiv 1(mod k), \lambda = k-1$. A near resolvable design is a special class of frame [see Ge and Miao (2007)].

Article History
Received : 18 January 2023; Revised : 07 May 2023; Accepted : 17 May 2023; Published : 30 June 2023

To cite this paper
Shyam Saurabh (2023). Some Methods of Constructions of Partially Balanced Designs. *Journal of Statistics and Computer Science*. 2(1), 47-53.

2.2 α – Resolvable design

Suppose b blocks of a block design $D(v,b,r,k)$ can be divided into $t = r/\alpha$ classes, each of size $\beta = v\alpha/k$ such that in each class of β blocks every treatment of D is replicated α times. Then these t classes are known as α –resolution (or parallel) classes and the design is called *α–resolvable design*. When $\alpha = 1$ the design is said to be resolvable and the classes are called resolution classes.

2.3 Balanced incomplete block design

A *balanced incomplete block design* (*BIBD*) or a $2-(v,k,\lambda)$ design is an arrangement of v treatments in b blocks, each of size $k(< v)$ such that every treatment occurs in exactly r blocks and any two distinct treatments occur together in λ blocks. The integers v, b, r, k, λ are called parameters of the BIBD and they satisfy the relations: $bk = vr; r(k-1) = \lambda(v-1), v \le b$ (Fisher's inequality).

Example 1: A near resolvable solution of a $2-(7,3,2)$ design is given below which may be found in Ge and Miao (2007):

Missing treatment	{0}	{1}	{2}	{3}	{4}	{5}	{6}
Blocks	{1, 2, 4} {3, 5, 6}	{2, 3, 5} {0, 4, 6}	{3, 4, 6} {0, 1, 5}	{0, 4, 5} {1, 2, 6}	{1, 5, 6} {0, 2, 3}	{0, 2, 6} {1, 3, 4}	{0, 1, 3} {2, 4, 5}

2.4 Group divisible design

Let $v = mn$ treatments be arranged in an $m \times n$ array. A *group divisible* (*GD*) *design* is an arrangement of the $v = mn$ treatments in b blocks each of size k such that:

1. Every treatment occurs at most once in a block;
2. Every treatment occurs in r blocks;
3. Every pair of treatments, which are in the same row of the $m \times n$ array, occur together in λ_1 blocks; while every other pair of treatments occur together in λ_2 blocks.

The integers $v = mn, b, r, k, \lambda_1$and λ_2 are known as parameters of the GD design and they satisfy the relations: $bk = vr; (n-1)\lambda_1 + n(m-1)\lambda_2 = r(k-1)$. Furthermore, if $r - \lambda_1 = 0$ then the GD design is singular (*S*); if $r - \lambda_1 > 0$ and $rk - v\lambda_2 = 0$ then it is semi–regular (*SR*); and if $r - \lambda_1 > 0$ and $rk - v\lambda_2 > 0$, the design is regular (*R*). For $\lambda_1 = 0, \lambda_2 = \lambda$, the above definition is equivalent to uniform $(k,\lambda)-$ GD design of type n^m, see Furino *et al.* (1996) and Abel *et al.* (2009).

2. 5 Triangular design

Let $v = n(n-1)/2$ treatments be arranged in an $n \times n$ array such that the positions on the principal diagonal are left blank, the $n(n-1)/2$ positions above and below the principal diagonal are filled with the v treatments in such a way that the resultant arrangement is symmetric about the principal diagonal. Then any two treatments which occur in the same row or same column of the array are first associates; otherwise they are second associates.

A partially balanced incomplete block design based on a triangular scheme is known as a *triangular design*. The integers $v = n(n-1)/2, b, r, k, \lambda_1$ and λ_2 are known as parameters of the triangular design and they satisfy the relations: $bk = vr; n_1\lambda_1 + n_2\lambda_2 = 2(n-2)\lambda_1 + \frac{(n-2)(n-3)}{2}\lambda_2 = r(k-1)$.

3. Constructions Methods

Theorem 1: The existence of λ – resolvable $(\lambda \ge 1)$ SRGD design with parameters:

$$v, b = r^2/\lambda, r, k, \lambda_1 = 0, \lambda_2 = \lambda, m, n = r/\lambda \quad (1)$$

implies the existence of another SRGD design with parameters:

$$v^* = v + n, b^* = b, r^* = r, k^* = k + 1, \lambda_1^* = 0, \lambda_2^* = \lambda, m^* = m + 1, n^* = n. \quad (2)$$

Proof: Since the SRGD design with parameters (1) is λ– resolvable, the number of resolution classes is $n = r/\lambda$ and the number of blocks in each resolution class is $b\lambda/r = v\lambda/k = r$. Let $R^1, R^2, \ldots, R^n$ be the resolution classes of SRGD design with parameters (1). Let $B_1^i, B_2^i, \ldots, B_r^i$ be arbitrarily chosen blocks in its i^{th} resolution class and $\theta_1, \theta_2, \ldots, \theta_n$ be the new treatments distinct from the v treatments of the SRGD design. We construct n new classes corresponding to n resolution classes of the SRGD design with parameters (2) as follows:

$B_1^1 \cup \{\theta_1\}$	$B_1^2 \cup \{\theta_2\}$	$\cdots$	$B_1^n \cup \{\theta_n\}$
$B_2^1 \cup \{\theta_1\}$	$B_2^2 \cup \{\theta_2\}$	$\cdots$	$B_2^n \cup \{\theta_n\}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$B_r^1 \cup \{\theta_1\}$	$B_r^2 \cup \{\theta_2\}$	$\cdots$	$B_r^n \cup \{\theta_n\}$

New treatments are added once only in each block of a resolution class. Clearly the block size is increased by one and the replication remains unchanged. Since the original SRGD design is λ–resolvable with $\lambda_1 = 0, \lambda_2 = \lambda$, any pair of distinct treatments which are first associates occur together in no block; and any pair of distinct treatments which are second associates occur together in λ blocks; also in the derived SRGD design. The union of these new classes generates the blocks of another SRGD design with parameters (2). The association scheme of the derived SRGD design is obtained by adjoining a new row: $mn + 1, mn + 2, \ldots, n(m + 1)$ to the $m \times n$ association scheme of the original SRGD design.

Example 2: Consider the following resolution classes of a 2–resolvable SRGD design *SR*36: $v = b = 8, r = k = 4, \lambda_1 = 0, \lambda_2 = 2, m = 4, n = 2$.

R^1	R^2
(1, 3, 6, 8)	(2, 3, 4, 5)
(1, 2, 3, 4)	(3, 5, 6, 8)
(4, 5, 6, 7)	(1, 4, 6, 7)
(2, 5, 7, 8)	(1, 2, 7, 8)

Then using Theorem 1, the blocks of *SR*52: $v = 10, b = 8, r = 4, k = 5, \lambda_1 = 0, \lambda_2 = 2, m = 5, n = 2$ are given as:

(1, 3, 6, 8, 9)	(2, 3, 4, 5, 10)
(1, 2, 3, 4, 9)	(3, 5, 6, 8, 10)
(4, 5, 6, 7, 9)	(1, 4, 6, 7, 10)
(2, 5, 7, 8, 9)	(1, 2, 7, 8, 10)

The 5 x 2 GD scheme is given as transpose of the array: $\begin{matrix} 1 & 2 & 3 & 4 & 9 \\ 5 & 6 & 7 & 8 & 10 \end{matrix}$.

TABLE 1: SRGD designs obtained using Theorem 1

No.	Original design	Derived design	No.	Original design	Derived design
1	*SR*36; 2– Resolvable	*SR*52	16	*SR*77; Resolvable	*SR*87
2	*SR*37;3– Resolvable	*SR*53	17	*SR*78; Resolvable	*SR*88
3	*SR*40; 5– Resolvable	*SR*55	18	*SR*79; Resolvable	*SR*89
4	*SR*43; 3–Resolvable	*SR*57	19	**SR*81; 3– Resolvable	*SR*91
5	*SR*48; Resolvable	*SR*62	20	*SR*82; 4– Resolvable	*SR*92
6	*SR*49; Resolvable	*SR*63	21	**SR*83; 5– Resolvable	*SR*93

7	$SR50$; Resolvable	$SR64$	22	$SR87$; Resolvable	$SR96$
8	$SR52$; 2– Resolvable	$SR66$	23	$SR88$; Resolvable	$SR97$
9	*$SR53$; 3– Resolvable	$SR67$	24	$SR89$; Resolvable	$SR98$
10	$SR60$; Resolvable	$SR75$	25	*$SR91$; 3– Resolvable	$SR99$
11	$SR62$; Resolvable	$SR77$	26	$SR98$; Resolvable	$SR105$
12	$SR63$; Resolvable	$SR78$	27	*$SR99$; 3– Resolvable	$SR106$
13	$SR64$; Resolvable	$SR79$	28	*$SR100$; 4– Resolvable	$SR07$
14	$SR66$; 2– Resolvable	$SR80$	29	$SR105$; Resolvable	$SR110$
15	*$SR67$; 3– Resolvable	$SR81$	30	*$SR70$; 5– Resolvable	$SR83$

*α– resolvable solutions are reported in Saurabh *et al.* (2021*b*).

Theorem 2: The existence of a near resolvable triangular design with parameters:

$$v^* = r+1, b^* = r(r+1)/k, r^*, k^*, n_1, n_2, \lambda_1, \lambda_2 \quad (3)$$

implies the existence of another triangular design with parameters:

$$v^{**} = r+1, b^{**} = b^*, r^{**} = r^* + r/k, k^{**} = k^* + 1, n_1, n_2, \lambda_1 + 2, \lambda_2 + 2. \quad (4)$$

Proof: Let $R^1, R^2, \ldots, R^v$ be the partial resolution classes of a near resolvable triangular design with parameters (3) and $\{\theta_i\}$ be the missing treatment in the i^{th} partial resolution class. Then the number of blocks in a partial resolution classes of a triangular design is $\alpha = r/k$. Let $B_1^i, B_2^i, \ldots, B_\alpha^i$ be the blocks in the i^{th} resolution class. Then the new blocks corresponding to R^i are obtained as: $B_1^i \cup \{\theta_i\}, B_2^i \cup \{\theta_i\}, \ldots, B_\alpha^i \cup \{\theta_i\}$. Then the replication of each treatment is increased by α, the block size is increased by one and any pair of distinct treatments which are first associates to each other occurs together in $\lambda_1 + 2$ blocks any pair of distinct treatments which are second associates to each other occurs together in $\lambda_2 + 2$ blocks. Hence we obtain a triangular design with parameters (4). The scheme of triangular design with parameters (4) is same as of the design with parameters (3).

Theorem 3: The existence of a near resolvable two–associate class cyclic design with parameters: $v^* = r+1, b^* = r(r+1)/k, r^*, k^*, n_1, n_2, \lambda_1, \lambda_2$ (5)

implies the existence of another cyclic design with parameters: $v^{**} = r+1, b^{**} = b^*, r^{**} = r^* + r/k, k^{**} = k^* + 1, n_1, n_2, \lambda_1 + 2, \lambda_2 + 2.$ (6)

Proof: As outlined in Theorem 2.

Example 3: A near resolvable solution of cyclic design C2: $v = 5, r = 4, k = 2, b = 10, n_1 = n_2 = 2, \lambda_1 = 2, \lambda_2 = 0$ is given below:

Missing Treatment	{1}	{2}	{3}	{4}	{5}
Blocks	{3, 5}	{3, 5}	{1, 4}	{1, 3}	{1, 3}
	{2, 4}	{1, 4}	{2, 5}	{2, 5}	{2, 4}

Then using Theorem 3 we obtain a cyclic design C13: $v = 5, r = 6, k = 3, b = 10, n_1 = n_2 = 2, \lambda_1 = 4, \lambda_2 = 2$ whose solution is given as:

{1, 3, 5}	{2, 3, 5}	{1, 3, 4}	{1, 3, 4}	{1, 3, 5}
{1, 2, 4}	{1, 2, 4}	{2, 3, 5}	{2, 4, 5}	{2, 4, 5}

4. Near resolvable triangular designs

By juxtaposing the blocks of solution number 1 of *T*9 and the blocks of solution number 3 of *T*10 as given in Clatworthy (1973), a new solution of *T*11 is obtained. This solution is used to construct a near resolvable solution. The triangular design *T*11 has parameters: $v = 10, r = 9, k = 3, b = 30, n_1 = 6, n_2 = 3, \lambda_1 = 3, \lambda_2 = 0.$

TABLE 2: A near resolvable triangular design *T*11 with 10 treatments

Missing Treatment	Partial Resolution Classes
{1}	{5, 6, 8}; {7, 9, 10}; {2, 3, 4}
{2}	{1, 3, 4}; {5, 8, 9}; {6, 7, 10}
{3}	{1, 2, 4}; {5, 7, 9}; {6, 8, 10}
{4}	{1, 2, 3}; {8, 9, 10}; {5, 6, 7}
{5}	{1, 3, 6}; {2, 8, 9}; {4, 7, 10}
{6}	{1, 2, 5}; {3, 8, 10}; {4, 7, 9}
{7}	{1, 5, 6}; {2, 3, 8}; {4, 9, 10}
{8}	{1, 5, 7}; {2, 4, 9}; {3, 6,10}
{9}	{1, 6, 7}; {2, 5, 8}; {3, 4, 10}
{10}	{1, 4, 7}; {2, 5, 9}; {3, 6, 8}

Using Theorem 2, we obtain a triangular design with parameters: $v = 10, r = 12, k = 4, b = 30, n_1 = 6, n_2 = 3, \lambda_1 = 5, \lambda_2 = 2$. The blocks are given as:

{1, 5, 6, 8}; {1, 7, 9, 10}; {1, 2, 3, 4}; {1, 2, 3, 4}; {2, 5, 8, 9}; {2, 6, 7, 10}; {1, 2, 3, 4};

{3, 5, 7, 9}; {3, 6, 8, 10}; {1, 2, 3, 4}; {4, 8, 9, 10}; {4, 5, 6, 7}; {1, 3, 5, 6}; {2, 5, 8, 9};

{4, 5, 7, 10}; {1, 2, 5, 6}; {3, 6, 8, 10}; {4, 6, 7, 9}; {1, 5, 6, 7}; {2, 3, 7, 8}; {4, 7, 9, 10}; {1, 5, 7, 8}; {2, 4, 8, 9}; {3, 6, 8, 10}; {1, 6, 7, 9}; {2, 5, 8, 9}; {3, 4, 9, 10}; {1, 4, 7, 10}; {2, 5, 9, 10}; {3, 6, 8, 10}.

A near resolvable solution for *T*13 is presented below. The triangular design *T*13 has parameters: $v = 10, r = 9, k = 3, b = 30, n_1 = 6, n_2 = 3, \lambda_1 = 1, \lambda_2 = 4$.

TABLE 3: A near resolvable triangular design *T*13 with 10 treatments

Missing Treatment	Partial Resolution Classes
{1}	{2, 5, 10}; {3, 6, 9}; {4, 7, 8}
{2}	{1, 8, 10}; {3, 5, 7}; {4, 6, 9}
{3}	{1, 9, 10}; {2, 7, 8}; {4, 5, 6}
{4}	{1, 8, 9}; {2, 6, 7}; {3, 5, 10}
{5}	{1, 4, 8}; {2, 6, 10}; {3, 7, 9}
{6}	{1, 3, 9}; {2, 7, 10}; {4, 5, 8}
{7}	{1, 2, 10}; {4, 6, 8}; {3, 5, 9}
{8}	{1, 6, 9}; {2, 3, 7}; {4, 5, 10}
{9}	{1, 5, 10}; {2, 4, 6}; {3, 7, 8}
{10}	{1, 7, 8}; {3, 4, 5}; {2, 6, 9}

Using Theorem 2 we obtain a triangular design with parameters: $v = 10, r = 12, k = 4, b = 30, n_1 = 6, n_2 = 3, \lambda_1 = 3, \lambda_2 = 6$. The blocks are given as:

{1, 2, 5, 10}; {1, 3, 6, 9}; {1, 4, 7, 8}; {1, 2, 8, 10}; {2, 3, 5, 7}; {2, 4, 6, 9}; {1, 3, 9, 10}; {2, 3, 7, 8}; {3, 4, 5, 6}; {1, 4, 8, 9}; {2, 4, 6, 7}; {3, 4, 5, 10}; {1, 4, 5, 8}; {2, 5, 6, 10};

{3, 5, 7, 9}; {1, 3, 6, 9}; {2, 6, 7, 10}; {4, 5, 6, 8}; {1, 2, 7, 10}; {4, 6, 7, 8}; {3, 5, 7, 9}; {1, 6, 8, 9}; {2, 3, 7, 8}; {4, 5, 8, 10}; {1, 5, 9, 10}; {2, 4, 6, 9}; {3, 7, 8, 9}; {1, 7, 8, 10}; {3, 4, 5, 10}; {2, 6, 9, 10}.

The arrangement of 10 treatments in a 5×5 array for the triangular designs *T*11 and *T*13 is given as:

$$\begin{array}{ccccc} * & 1 & 2 & 3 & 4 \\ 1 & * & 5 & 6 & 7 \\ 2 & 5 & * & 8 & 9 \\ 3 & 6 & 8 & * & 10 \\ 4 & 7 & 9 & 10 & * \end{array}.$$

5. Conclusion

A series of near resolvable triangular or cyclic designs is still not known in the literature. Here some examples of such designs are given. A pair of orthogonal near resolutions of triangular/cyclic designs may also be investigated. Such types of designs are useful in anonymous $(2, k)$ −threshold schemes and in unconditionally secure commitment schemes [see Topalova and Zhelezova (2009)].

Further a recursive construction of SRGD designs is given. Some recursive constructions may also be found in Banerjee and Kageyama (1996) and Saurabh and Sinha (2022). The GD designs are useful in group testing experiments, sampling and intercropping [see Raghavarao and Padgett (2005)] and a GD design with $\lambda_1 = 0, \lambda_2 = 1$ is useful in the construction of low–density parity–check (LDPC) codes free of 4–cycles and key pre–distribution schemes [see Lee and Stinson (2005) and Xu *et al.* (2019)].

Acknowledgement

The author is thankful to anonymous reviewers, Editor–in–Chief and Dr. Kishore Sinha for their valuable suggestions in improving the presentation and readability of the paper.

References

Abel, R. J. R., Lamken, E. R. and Wang, J. (2008). A few more Kirkman squares and doubly near resolvable BIBDs with block size 3. *Discrete Mathematics*, 308, 1102–1123.

Abel, R. J. R., Combe, D., Price, G. and Palmer, W. D. (2009). Existence of generalized Bhaskar Rao designs with block size 3. *Discrete Mathematics*, 309, 4069–4078.

Abel, R. J. R. (2017). Existence of near resolvable $(v, k, k - 1)$ BIBDs with $k \in \{9, 12, 16\}$. *Australasian Journal of Combinatorics,* 67(1), 25–45.

Arasu, K. T., Bansal, P. and Watson, C. (2013). Partially balanced incomplete block designs with two associate classes. *Journal of Statistical Planning and Inference*, 143, 983–991.

Bagchi, S. (2004). Construction of group divisible designs and rectangular designs from resolvable and almost resolvable balanced incomplete block designs. *Journal of Statistical Planning and Inference*, 119, 401–410.

Banerjee, S. and Kageyama, S. (1996). Non–isomorphic solutions of group divisible designs with $k = 3$ and $\lambda_1 = 0$. *Journal of Statistical Planning and Inference*, 56, 23–32.

Bassalaygo, L. A. and Zinoviev, V. A. (2016). On resolvable and near resolvable BIB designs and q −array equidistant codes. *Fifteen International Workshop on Algebraic and Combinatorial Coding Theory*, June 18–24, Albena, Bulgaria, 41–46.

Clatworthy, W. H. (1973). *Tables of Two–Associate–classes partially balanced designs, National Bureau of Standards.* Applied Mathematics Series 63, Washington, D. C.

Ge, G. and Miao, Y. (2007). PBDs, Frames, and Resolvability. Colbourn, C. J. and Dinitz, J. H. (editors). Handbook of Combinatorial designs, Second edition, Chapman & Hall/ CRC, New York, 261–265.

Dey, A. (2010). Incomplete block designs. Hindustan Book Agency, New Delhi.

Furino, S. (1995). Existence results for near resolvable designs. *Journal of Combinatorial Designs*, 3, 101–113.

Furino, S., Miao, Y. and Jianxing, Y. (1996). Frames and Resolvable Designs: Uses, Constructions and Existence. CRC Press, New York.

Lee, J. and Stinson, D. (2005). A combinatorial approach to key pre–distribution for distributed sensor networks. In IEEE *wireless communications and networking conference (WCN' 05), IEEE communication society*, 1200–1205.

Raghavarao, D. (1988). Constructions and combinatorial problems in design of experiments. Dover Publications, New York.

Raghavarao, D. and Padgett, Lakshmi V. (2005). Block designs. Analysis, Combinatorics and Applications, Series on Applied Mathematics. World Scientific, Singapore.

Saurabh, S. (2022). Combinatorial matrices from certain balanced incomplete block designs. *Journal of Statistics and Computer Science.* 1(2), 159–164.

Saurabh, S., Sinha, K. and Singh, M. K. (2021*a*). Unifying constructions of group divisible designs. *Statistics and Applications*, 19 (1), 125–140.

Saurabh, S., Sinha, K. and Singh, M. K. (2021*b*). A survey of resolvable solutions of partially balanced designs, *Communications in Statistics–Theory and Methods*, DOI: 10.1080/03610926.2021.1942049.

Saurabh, S. and Sinha, K. (2022). Some recursive constructions of α– resolvable group divisible designs. *Statistics and Applications*, 20 (1), 141–148.

Saurabh, S. and Sinha, K. (2023). Matrix approaches to constructions of group divisible designs. *Bulletin of the ICA*, 97, 83–105.

Topalova, S. and Zhelezova, S. (2009). Classification of doubly resolvable designs and orthogonal resolutions. B. Preneel *et al.* (Eds.). Enhancing cryptographic primitives with techniques from error correcting codes, IOS Press, 260–264.

Trung, van T. (1999). Constructions for near resolvable designs and BIBDs. *Journal of Combinatorial Designs*, 7(3), 227–231.

Xu, H., Yu, Z., Feng, D. and Zhu, H. (2019). New construction of partial geometries based on group divisible designs and their associated LDPC codes. *Physical Communication*, https://doi.org/10.1016/j.phycom.2019.100970.